Tutorial : Linear Optimization

GIAN Short Course on Optimization: Applications, Algorithms, and Computation

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The Busy College Student Problem

Formulate the student problem in AMPL (see Lecture 10)

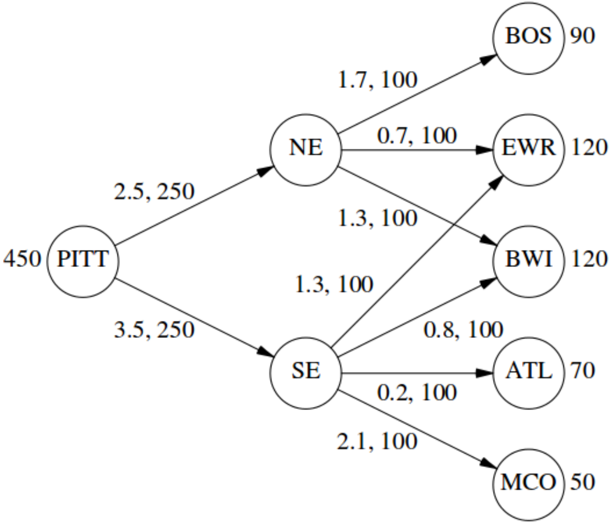
- $h(t) \geq 0$ as hours spent on task $t \in \mathcal{T}$
- maximize value $\underset{h}{\text{maximize}} \quad \sum_{t \in \mathcal{T}} \text{Value}(t) \cdot h(t)$
- College Rule 1: $h(\text{study}) + h(\text{tutorial}) \leq h(\text{lecture})$
- College Rule 2: $h(\text{study}) + h(\text{tutorial}) + h(\text{lecture}) \geq 8$
- College Rule 3: $h(\text{study}) + \frac{3}{2} h(\text{tutorial}) + 2 h(\text{lecture}) \geq 10$
- Parents Rule 1: $h(\text{eat}) + h(\text{sleep}) \geq 10$
- Parents Rule 2: $h(\text{sleep}) \geq 8 h(\text{eat})$
- Only 24 hours: $\sum_{t \in \mathcal{T}} h(t) \leq 24$

... you can add your own preferences!

Tutorial : A transshipment model

- A plant PITT makes 450 packs of a product. Cities NE and SE are northeast and southeast distribution centers (DC).
- The DCs receive packs from PITT and ship them to warehouses at cities BOS, EWR, BWI, ATL and MCO.
- For each intercity ‘link’ there is shipping cost per pack and an upper limit on the packs that can be shipped (shown in figure).
- Find the lowest-cost shipping plan of packs over available links, respecting the specified capacities and meeting the demands at warehouses. Use network.dat for input data.

Tutorial : The network



Tutorial : Mathematical model

Notation

- Set of all cities: $\mathcal{C}$
- Set of all links between cities: $\mathcal{L}$
- Supply from city k: s_k
- Demand at city k: d_k
- Cost of transshipment from city i to j: c_{ij}
- Capacity of link (i,j): U_{ij}
- Amount of packs to be transfered from city i to j: x_{ij}

$$\begin{aligned} \underset{x}{\text{minimize}} \quad & \sum_{i,j \in \mathcal{C} : (i,j) \in \mathcal{L}} c_{ij} x_{ij} && \text{(objective)} \\ \text{.subject to:} \quad & s_k + \sum_{(i,k) \in \mathcal{L}} x_{ik} \geq d_k + \sum_{(k,j) \in \mathcal{L}} x_{kj}, \ \forall k \in \mathcal{C} && \text{(balance cons.)} \\ & 0 \leq x_{ij} \leq U_{ij}, \quad \forall i,j \in \mathcal{C} : (i,j) \in \mathcal{L} && \text{(bound cons.)} \end{aligned}$$

Exercise: QP in Portfolio Selection

Problem Data

- $n = 4$ number of available assets
- $r = 1000$ desired minimum growth of portfolio
- $\beta = 10000$ available capital for investment
- m_i expected rate of return of asset i
  $m_1 = 0.5, m_2 = -0.2, m_3 = 0.15, m_4 = 0.30.$
- Covariance matrix of asset returns $C =$
  $$\begin{bmatrix} 0.08 & -0.05 & -0.05 & -0.05 \\ -0.05 & 0.16 & -0.02 & -0.02 \\ -0.05 & -0.02 & 0.35 & 0.06 \\ -0.05 & -0.02 & 0.06 & 0.35 \end{bmatrix}$$

Problem Variables

- $x_i \geq 0$ amount of investment in asset i
- Assume $x_i \geq 0$ and $x_i \in \mathbb{R}$ real

Exercise: QP in Portfolio Selection

Problem Objective

- Minimize risk of investment

$$\underset{x}{\text{minimize}} \quad x^T C x$$

Problem Constraints

- Minimum rate of return on investment

$$\sum_{i=1}^{n} m_i x_i \geq r$$

- Upper bound on total investment

$$\sum_{i=1}^{n} x_i \leq \beta$$