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Oscillation Mitigation in Electric Power Grids via Semidefinite Programming

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Abstract— **Inter-area oscillation of angle differences between groups of power generators is considered the precursor of large-scale blackouts. Because of the wide-area interconnection of generators, merely attenuating oscillation at individual sites is insufficient. In this paper, we consider the design of generator parameters and the interconnection strength such that the spectrum of the power system is assigned to a prespecified region of the complex plane. For identical generators, we obtain an explicit relation between the spectrum of the linearized dynamics and the spectrum of the interconnection matrix. This result allows us to formulate the eigenvalue problem as a semidefinite program, which can be solved efficiently. We provide an example to demonstrate the effectiveness of our approach.**

Keywords: Dynamic stability, oscillation, power systems, semidefinite programming, swing equations.

I. Dynamic stability in power systems

Stability is well recognized as the essential part of power system operation. Traditionally, three types of power system stability are studied: steady-state, dynamic, and transient stability [1]. In this paper, we focus on dynamic stability, which is concerned with the oscillatory behavior of power systems under small disturbances [2]. While oscillations associated with a single generator is well understood, oscillations associated with groups of generators that span a wide area is still not fully understood [3]. Many believe that growing inter-area oscillations can result in catastrophic consequences such as the 1996 Western blackout [4].

Research efforts have focused on detecting, monitoring, and suppressing wide-area oscillations (see [1]–[5] and the references therein). In [3], detailed experiments are conducted to study the effects of system components such as line impedance, load characteristics, and excitation signals on inter-area oscillation. In [4], several signal-processing engines based on synchrophasor measurements are developed for oscillation monitoring. In [5], a comprehensive study of power system oscillations via eigenvalue sensitivity is conducted. Furthermore, various control strategies ranging from redispatch to voltage adjustments are developed and validated against several test cases [5].

The fundamental mathematical tool for oscillation studies is modal analysis of power systems. Eigenvalues and eigenvectors of system dynamics have been proved to encode valuable information for oscillations. Therefore, most research work centers on eigen-analysis of a given power system. In this paper, we consider the parameter design of generators and their interconnection strength to mitigate inter-area oscillations. For groups of identical generators, we show that the eigenmodes of oscillation are determined by the spectrum of the interconnection matrix. Armed with this result, we propose a semidefinite programming (SDP) approach that assigns eigenvalues within a desired region.

The rest of the paper is organized as follows. In Section II, we present the linearized model for generator dynamics. In Section III, we perform a spectrum analysis and propose a region of performance oscillation mitigation. In Section IV, we provide an algorithm for eigenvalue assignment based on semidefinite programming. In Section V, we show the mitigation of oscillation via an example. In Section VI, we conclude the paper with a brief summary.

II. Linearized Swing equation

Following [6], we consider a network of N generators. The motion of generator i can be modeled as [1]

$$M_i \ddot{\theta}_i + D_i \dot{\theta}_i = P_{m,i} - P_{e,i},$$

where M_i is the inertia, D_i is the damping, θ_i is the rotor angle, and $P_{m,i}$ and $P_{e,i}$ are the mechanical and electrical power, respectively. The electrical power of generator i is given by [7]

$$P_{e,i} = g_{ii}|V_i|^2 + \sum_{i \sim k} g_{ik} |V_i||V_k| \cos(\theta_i - \theta_k) + \sum_{i \sim k} b_{ik} |V_i||V_k| \sin(\theta_i - \theta_k),$$

where V_i and g_{ii} are the voltage and the self-admittance of generator i, respectively, and g_{ik} and b_{ik} are the conductance and susceptance of the line connecting

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generators i and k, respectively. Under the standard assumptions [7] that conductances are negligible (i.e., $g_{ik} \approx 0$), angle differences are small (i.e., $|\theta_i - \theta_k| \ll 1$), and voltages are constant with unit magnitude (i.e., $|V_i| = 1$), we obtain the linearized swing equation [2]

$$M_i \ddot{\theta}_i + D_i \dot{\theta}_i + \sum_{i \sim k} b_{ik}(\theta_i - \theta_k) = P_{m,i}. \quad (1)$$

The state-space representation of (1) is given by

$$\begin{bmatrix} \dot{\theta} \\ \dot{\omega} \end{bmatrix} = \begin{bmatrix} 0 & I \\ -M^{-1}L & -M^{-1}D \end{bmatrix} \begin{bmatrix} \theta \\ \omega \end{bmatrix} + \begin{bmatrix} 0 \\ I \end{bmatrix} P_m, \quad (2)$$

where $\omega = \dot{\theta}$ is the angular velocity, I is the identity matrix, and

$$M = \text{diag}\{M_i\}_i^n, \quad D = \text{diag}\{D_i\}_i^n$$

are diagonal matrices. We note that the interconnection of generators is encoded in matrix L, which is also known as the graph Laplacian matrix.

III. Spectrum constraints

The oscillatory behavior of the system is determined by the eigenvalues of matrix

$$A = \begin{bmatrix} 0 & I \\ -M^{-1}L & -M^{-1}D \end{bmatrix}. \quad (3)$$

Let $\gamma = \gamma_{\text{real}} + \mathbf{j}\gamma_{\text{imag}}$ be an eigenvalue of A. Note that A has a zero eigenvalue irrespective of the choice of inertia and damping coefficients, that is, M and D. The reason is that the Laplacian matrix L has a zero eigenvalue associated with the vector of all ones [8]

$$L\mathbf{1} = 0 \cdot \mathbf{1}.$$

We can thus readily verify that $v = [\mathbf{1}^T \; \mathbf{0}^T]^T$ is an eigenvector of A associated with $\gamma = 0$.

Since the decay rate associated with an eigenmode γ is determined by the real part γ_{real} and since the oscillation frequency is determined by the imaginary part γ_{imag}, we are interested in assigning the eigenvalues of A within a suitable performance region, for example,

$$\underline{R} \leq \gamma_{\text{real}} \leq \bar{R} < 0, \qquad \left|\frac{\gamma_{\text{imag}}}{\gamma_{\text{real}}}\right| \leq \bar{C}. \quad (4)$$

Here, the upper bound $\bar{R}$ keeps the spectrum of A from the imaginary axis, which can be considered as a measure of the stability margin. The lower bound $\underline{R}$ prevents trivial solutions that push the spectrum far left into the complex plane. The upper bound $\bar{C}$ on the ratio between imaginary and real parts determines the angle of the sector in the complex plane (e.g., see Fig. 2).

However, constraints on the spectrum of a generic A-matrix in (3) are hard constraints. The reason is that eigenvalues of a nonsymmetric matrix are typically nonconvex and nonsmooth functions of the matrix elements [9], [10].

Following [5], we assume that all generators are identical. Therefore,

$$M_i = \alpha, \quad D_i = \beta, \quad \text{for } i = 1, \ldots, N. \quad (5)$$

In this case, we can derive an explicit relation between the spectrum of A and the spectrum of the interconnection matrix L.

Proposition 3.1: Let γ and λ be an eigenvalue of A and L, respectively. Under condition (5), we have

$$\gamma = \frac{1}{2\alpha}(-\beta \pm \sqrt{\beta^2 - 4\alpha\lambda}). \quad (6)$$

Proof: From the definition of eigenvalues of A,

$$(A - \gamma I)v = -\begin{bmatrix} \gamma I & -I \\ (1/\alpha)L & (\gamma + \beta/\alpha)I \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = 0,$$

it follows that

$$\begin{aligned} v_2 &= \gamma v_1 \\ Lv_1 &= -(\gamma\alpha + \beta)v_2. \end{aligned}$$

Eliminating v_2 yields that

$$Lv_1 = -(\alpha\gamma^2 + \beta\gamma)v_1.$$

The solution of the quadratic equation

$$\alpha\gamma^2 + \beta\gamma + \lambda = 0$$

is given by (6), which completes the proof. ■

Proposition 3.1 says that γ has a nonzero imaginary part if and only if

$$\lambda > \beta^2/(4\alpha). \quad (7)$$

When (7) holds, all complex eigenvalues share the same real part, that is,

$$\gamma_{\text{real}} = -\beta/(2\alpha).$$

IV. Eigenvalue placement via semidefinite programming

Given a network of identical generators, we wish to design system parameters, namely, the inertia α, the damping β, and the susceptance b_{ik} over all lines such that the spectrum of A resides in the region of performance (4). We next devise a strategy in Algorithm 1 and prove its correctness in Proposition 4.1.

Proposition 4.1: The solution α, β, and L from Algorithm 1 results in matrix A in (3) whose spectrum resides in region (4) except for the origin.

Proof: Let

$$0 = \lambda_1 < \lambda_2 \leq \cdots \leq \lambda_N$$

be the eigenvalues of L. From (6), it follows that the pair $\gamma_{1,\pm}$ associated with $\lambda_1 = 0$

$$\gamma_{1,\pm} = \frac{1}{2\alpha}(-\beta \pm \sqrt{\beta^2 - 4\alpha\lambda_1})$$

Algorithm 1 Assignment of spectrum of A via semidefinite programming

1: Choose α, $\beta > 0$ such that

$$\underline{R} \leq -\frac{\beta}{\alpha} \leq -\frac{\beta}{2\alpha} \leq \bar{R}. \tag{8}$$

2: Solve the following semidefinite program

$$\begin{array}{ll} \text{minimize} & (1/2)\,\text{trace}\,(L) = \sum_{i\sim k} b_{ik} \\ \text{subject to} & L + c_2 J \succeq c_2 I \\ & L \preceq c_N I \end{array} \tag{9}$$

where $c_2 = \beta^2/(4\alpha)$, $c_N = \beta^2(1+\bar{C}^2)/(4\alpha)$, and $J = (1/N)\mathbf{1}\mathbf{1}^T$, that is, a matrix with all elements equal to $1/N$.

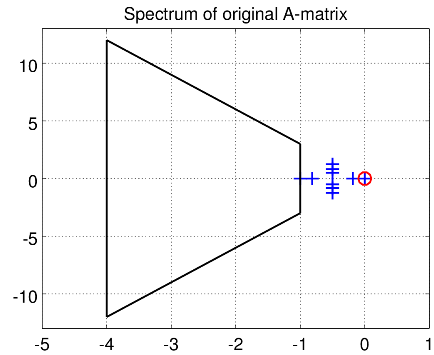

Fig. 2. The spectrum of the original system does not reside in the region of performance.

is given by $\gamma_{1,+} = 0$ and $\gamma_{1,-} = -\beta/\alpha$. From condition (8), we have

$$\underline{R} \leq \gamma_{1,-} \leq \bar{R}.$$

Since $J = (1/N)\mathbf{1}\mathbf{1}^T$ spans the nullspace of L, it follows that the spectrum of $L + c_2 J$ is the union of c_2 and $\{\lambda_i\}$ for $i = 2, \ldots, N$. From the SDP-constraints (9), we have

$$L + c_2 J \succeq c_2 I \;\Rightarrow\; \lambda_2 \geq c_2$$

and

$$L \preceq c_N I \;\Rightarrow\; \lambda_N \leq c_N.$$

Since $\lambda_2 \geq \beta^2/(4\alpha)$, it follows that $\gamma_{i,\pm}$ are complex eigenvalues and their real parts are equal to $-\beta/(2\alpha) \in [\underline{R}, \bar{R}]$. It remains to show that the ratio between imaginary and real parts are bounded by $\bar{C}$, that is,

$$\left| \frac{\sqrt{4\alpha\lambda_i - \beta^2}}{\beta} \right| \leq \bar{C}. \tag{10}$$

With some algebra, (10) follows from

$$\frac{\beta^2}{4\alpha} \leq \lambda_i \leq \frac{\beta^2(1+\bar{C}^2)}{4\alpha}. \quad i = 2, \ldots, N.$$

■

V. Example

We consider a series connection of $N = 5$ generators shown in Fig. 1. The inertia and the damping coefficients are $M_i = D_i = i$ for $i = 1, \ldots, N$. We set $b_{ik} = 1$ for all lines $i \sim k$. Figure 2 shows the eigenvalues of A. Note that an eigenvalue sits at the origin because of the zero eigenvalue of L. This is fact is independent of the design parameters M_i, D_i, and b_{ik}. We are interested in assigning spectrum of A within the region of performance (4) where $\underline{R} = -4$, $\bar{R} = -1$, and $\bar{C} = 3$.

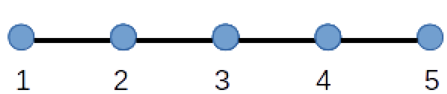

Fig. 1. Series connection of 5 generators.

For identical generators with $M_i = \alpha$ and $D_i = \beta$ for $i = 1, \ldots, N$, we choose $\alpha = 1$ and $\beta = 3$ to satisfy (8). We use YALMIP [11] to solve the SDPs in Algorithm 1. The solution from the SDP in (9) is given by $b_{12} = b_{45} = 4.50$ and $b_{23} = b_{34} = 6.75$. The total inertial, damping, and susceptance of the designed system are 5, 15, and 22.5, respectively, as compared with 15, 15, and 4 of the original system.

Figure 3 shows that the spectrum of the designed system resides within the desired region. We inject a disturbance at generator 1,

$$P_{m,1}(t) = e^{-\sigma t}\sin(wt), \tag{11}$$

where $\sigma = -0.5$ and $w = 4.1$.

Figure 5 and Figure 6 show the angle difference $\theta_i - \theta_1$ for $i = 2, \ldots, N$ for the original system and the designed system, respectively. We note that the oscillation of the original system is mitigated in both amplitude and time length. In particular, the peak amplitude for the original system is 0.17 which is twice of the peak amplitude of the designed system. On the other hand, the oscillation length for the design system is around 10 seconds which is half of the oscillation length for the original system.

VI. Conclusions

In this paper, we design parameters of generators and their interconnection to mitigate wide-area oscillation induced by disturbances. We propose a region of performance for the spectrum of the designed system. We show that for identical generators this problem can be solved via semidefinite programming. An example shows the effectiveness of our approach for the oscillation mitigation.

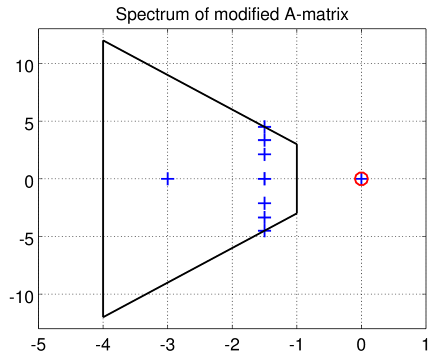

Fig. 3. The spectrum of the designed system with identical generators resides in the desired region.

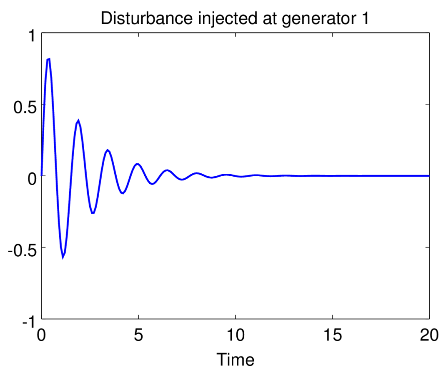

Fig. 4. Short-time disturbance input.

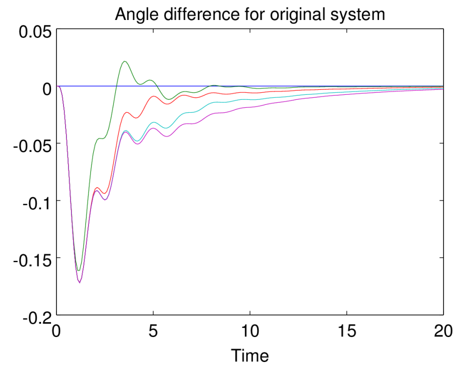

Fig. 5. Phase angle difference of the original system whose spectrum is shown in Figure 2.

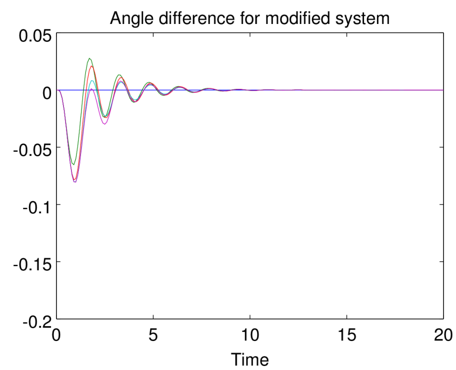

Fig. 6. Phase angle difference of the designed system whose spectrum is shown in Figure 3.

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