Tutorial 8: Perspective Reformulation

GIAN Short Course on Optimization: Applications, Algorithms, and Computation

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Uncapacitated Facility Location Problem (UFL)

Problem description:

- Set of facilities $\mathcal{I}$, and a set of customers $\mathcal{J}$.
- Customers have unit demand, which is met from open facilities.
- Shipment cost q_{ij} from facility $i \in \mathcal{I}$ to customer $j \in \mathcal{J}$ is proportional to square of the quantity shipped.
- Fixed charge c_i if facility $i \in \mathcal{I}$ is open, otherwise 0.
- Minimize total cost: sum of fixed cost and shipment cost.

Decision variables:

- $z_i \in \{0, 1\}$: Binary $= 1$ if facility $i \in \mathcal{I}$ open, $= 0$otherwise.
- x_{ij}: Quantity shipped from facility $i \in \mathcal{I}$ to customer $j \in \mathcal{J}$.

Uncapacitated Facility Location Problem(UFL)

Objective: Minimize total cost

$$\text{Min} \sum_{i \in \mathcal{I}} c_i z_i + \sum_{i \in I} \sum_{j \in J} q_{ij} x_{ij}^2,$$

Constraints

1. Quantity can be shipped only from open facilities

$$x_{ij} \leq z_i, \ \forall i \in \mathcal{I}, j \in \mathcal{J},$$

2. Demand satisfaction of customers

$$\sum_{i \in \mathcal{I}} x_{ij} = 1, \quad \forall j \in \mathcal{J},$$

3. Non-negativity constraints

$$z_i \in \{0, 1\}, \ x_{ij} \ \geq 0, \ \forall i \in \mathcal{I}, j \in \mathcal{J}.$$

Uncapacitated Facility Location Problem(UFL)

Reformulate model using auxiliary variables y_{ij}, $i \in \mathcal{I}$, $j \in \mathcal{J}$.

$$\text{Min} \sum_{i \in \mathcal{I}} c_i z_i + \sum_{i \in \mathcal{I}} \sum_{j \in \mathcal{J}} q_{ij} y_{ij} \tag{1}$$

$$\text{Subject to: } x_{ij} \leq z_i, \qquad \forall i \in \mathcal{I}, j \in \mathcal{J}, \tag{2}$$

$$\sum_{i \in \mathcal{I}} x_{ij} = 1, \qquad \forall j \in \mathcal{J}, \tag{3}$$

$$\frac{x_{ij}^2}{z_i} \leq y_{ij}, \qquad \forall i \in \mathcal{I}, j \in \mathcal{J}, \tag{4}$$

$$0 \leq z_i \leq 1, \; x_{ij} \geq 0, \qquad \forall i \in \mathcal{I}, j \in \mathcal{J}. \tag{5}$$

Uncapacitated Facility Location Problem(UFL)

Consider the following different formulations of constraint (4):

- $F1 : x_{ij}^2 <= y_{ij} * z_i, \quad \forall i \in \mathcal{I}, j \in \mathcal{J}$
- $F2 : x_{ij}^2 / z_i <= y_{ij}, \quad \forall i \in \mathcal{I}, j \in \mathcal{J}$
- $F3 : \sqrt{(4 * x_{ij}^2 + (y_{ij} - z_i)^2)} <= y_{ij} + z_i, \quad \forall i \in \mathcal{I}, j \in \mathcal{J}$
- $F4 : \dfrac{x_{ij}^2}{(z_i + 0.0001)} <= y_{ij}, \quad \forall i \in \mathcal{I}, j \in \mathcal{J}$
- $F5 : \dfrac{x_{ij}^2}{((1 - 0.0001) * z_i + 0.0001)} <= y_{ij}, \quad \forall i \in \mathcal{I}, j \in \mathcal{J}$

Implement all the five models in AMPL and run each of the models with different solvers, and report the outcome.

AMPL Modeling Tip 1: Named Problems

```
var x >=0, := 1;
var y, := 1;
var z binary, := 0.5;

minimize cost: z + y;

subject to
  linear: x <= z;

  # ... different formulations of perspective
  F1: x^2 <= y*z;
  F2: x^2/z <= y;
  F3: x^2 <= y*z;

# ... define a problem for each formulation
problem Formulation1: x, y, z, cost, linear, F1;
problem Formulation2: x, y, z, cost, linear, F2;
problem Formulation3: x, y, z, cost, linear, F3;
```

AMPL Modeling Tip 2: Run Files

```
printf "**********************************************\n";
printf "Solving UFL Formulation F2 by Solver snopt ...\n";
printf "**********************************************\n";
reset;
option solver snopt;
model UFL.mod;
solve Formulation2 > UFL_F2_snopt.out;
display _varname, _var >> UFL_F2_snopt.out;
display _conname, _con >> UFL_F2_snopt.out;
display _varname, _var;
```

See `UFL.mod` and `UFL.ampl`

Usage

```
ampl: model UFL.ampl;
```

Delete all the temp files that yuou generate at the end!