Tutorial 10: Mixed-Integer Nonlinear Optimization

GIAN Short Course on Optimization: Applications, Algorithms, and Computation

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Tutorial 10: Mixed-Integer Nonlinear Optimization

1. Write an AMPL model of the following MINLP, and solve it.

$$
\begin{aligned}
\underset{x,y}{\text{minimize}} \quad & 5y_1 + 6y_2 + 8y_3 + 10x_1 - 7x_3 \\
& -18\log(x_2 + 1) - 19.2\log(x_1 - x_2 + 1) \\
\text{subject to} \quad & 0.8\log(x_2 + 1) + 0.96\log(x_1 - x_2 + 1) - 0.8x_3 \geq 0 \\
& \log(x_2 + 1) + 1.2\log(x_1 - x_2 + 1) - x_3 - 2y_3 \geq -2 \\
& x_2 - x_1 \leq 0 \\
& x_2 - 2y_1 \leq 0 \\
& x_1 - x_2 - 2y_2 \leq 0 \\
& y_1 + y_2 \leq 1 \\
& y \in \{0,1\}^3,\ x \geq 0, x_1,\ x_2 \leq 2,\ x_3 \leq 1
\end{aligned}
$$

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2. Consider the AMPL model `water-net.mod` and improve it.
   - Get the model and data files from our course webs-site and solve them using `knitro` (`baron` would be slow).
   - Improve the AMPL model: (1) make the diameters discrete

     $$d_{i,j} \in \{0.25, 0.5, 1.0, 2.0\} \quad \forall (i,j) \in \mathcal{A}$$

     using SOS-1; (2) introduce area variables, $a_{i,j}$, $(i,j) \in \mathcal{A}$, and linearize the diameter bound.
   - Consider replacing the binary variables that model flow direction, `z[i,j]`, by a complementarity constraint on `qp[i,j]` and `qn[i,j]`, see lecture.

   Document how each change affects the optimal solution value and solve time!

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3. Assume $c(x) \leq 0$ convex and $\mathcal{C}^2$, and $\exists i : c_i(\hat{x}) > 0$. Show that $\hat{x}$ violates
$$0 \geq \hat{c}_i + \nabla \hat{c}^T (x - \hat{x}).$$

4. Consider the worst-ever nonlinear function,
$$z = \frac{1}{1 + 1000(x - y)^{10}} \approx \begin{cases} 1 & \text{if } x = y \\ 0 & \text{otherwise} \end{cases}$$
which "models" that $z = 1$, if $x = y$, and $z = 0$, if $x \neq y$. Assuming that $0 \leq x, y \leq U$ are integers, derive an equivalent linear model using SOS.