Tutorial 7: Nonlinear Optimization

GIAN Short Course on Optimization: Applications, Algorithms, and Computation

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Barrier Methods and Looping in AMPL

This exercise shows you how to use loops in AMPL

Use the example from the lectures, namely

$$\underset{x}{\text{minimize}}\ x_1^2 + x_2^2 \quad \text{subject to } x_1 + x_2^2 - 1 \geq 0$$

and choose $x^{(0)} = (2,2)^T$ as your starting point
... making sure the barrier term is defined!

- Write an AMPL model that implements the barrier problem
- Loop over the barrier parameter, reducing it from 1 to 10^{-4}
- Display the results

The syntax for an AMPL for-loop is:

```
for{i in 1..5}{
   solve;    # ... solves the SAME problem 5 times
}; # end for
```

Exercise: QP in Portfolio Selection

Problem Data

- $n = 4$ number of available assets
- $r = 1000$ desired minimum growth of portfolio
- $\beta = 10000$ available capital for investment
- m_i expected rate of return of asset i
  $m_1 = 0.5, m_2 = -0.2, m_3 = 0.15, m_4 = 0.30.$
- Covariance matrix of asset returns $C =$
  $$\begin{bmatrix} 0.08 & -0.05 & -0.05 & -0.05 \\ -0.05 & 0.16 & -0.02 & -0.02 \\ -0.05 & -0.02 & 0.35 & 0.06 \\ -0.05 & -0.02 & 0.06 & 0.35 \end{bmatrix}$$

Problem Variables

- $x_i \geq 0$ amount of investment in asset i
- Assume $x_i \geq 0$ and $x_i \in \mathbb{R}$ real

Exercise: QP in Portfolio Selection

Problem Objective

- Minimize risk of investment

$$\underset{x}{\text{minimize}} \quad x^T C x$$

Problem Constraints

- Minimum rate of return on investment

$$\sum_{i=1}^{n} m_i x_i \geq r$$

- Upper bound on total investment

$$\sum_{i=1}^{n} x_i \leq \beta$$

Exercise: Sparse Optimization

A problem arising in sparse optimization or compressive sensing is

$$\underset{x}{\text{minimize}} \ \|Ax - b\|_2^2 + \lambda \|x\|_1$$

the regularized least-squares problem.

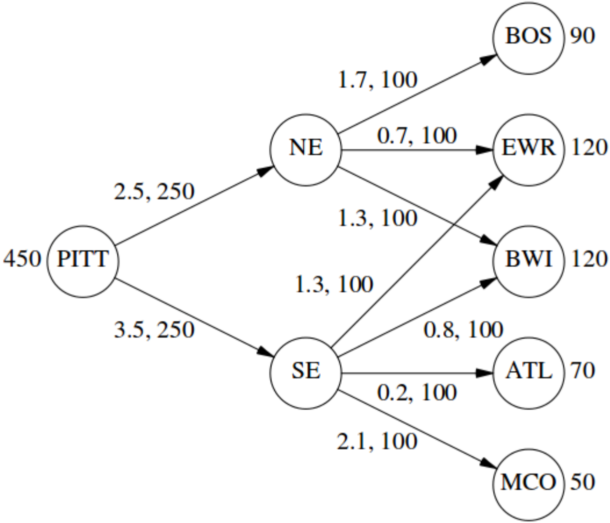
1. Formulate this problem as a smooth QP (see lectures)
2. Create an AMPL model of the smooth problem
3. Use the data file `SparseOpt.dat` provided
   ... from Michael Friedlander's `spgl1` Matlab tools
4. Solve the problem for $\lambda \in \{0.01, 0.1, 1, 2, 4, 8, 16\}$
   ... using looping of course
5. Record the values of $\|Ax - b\|_2^2$ and $\|x\|_1$ for each λ

Hint: Write separate `*.mod`, `*.dat`, and `*.ampl` files

Tutorial : A transshipment model

- A plant PITT makes 450 packs of a product. Cities NE and SE are northeast and southeast distribution centers (DC).
- The DCs receive packs from PITT and ship them to warehouses at cities BOS, EWR, BWI, ATL and MCO.
- For each intercity ‘link’ there is shipping cost per pack and an upper limit on the packs that can be shipped (shown in figure).
- Find the lowest-cost shipping plan of packs over available links, respecting the specified capacities and meeting the demands at warehouses. Use network.dat for input data.

Tutorial : The network



Tutorial : Mathematical model

Notation

- Set of all cities: $\mathcal{C}$
- Set of all links between cities: $\mathcal{L}$
- Supply from city k: s_k
- Demand at city k: d_k
- Cost of transshipment from city i to j: c_{ij}
- Capacity of link (i,j): U_{ij}
- Amount of packs to be transfered from city i to j: x_{ij}

$$
\begin{array}{llr}
\underset{x}{\text{minimize}} & \displaystyle\sum_{i,j\in\mathcal{C}:(i,j)\in\mathcal{L}} c_{ij}x_{ij} & \text{(objective)} \\
.\text{subject to:} & s_k + \displaystyle\sum_{(i,k)\in\mathcal{L}} x_{ik} \geq d_k + \sum_{(k,j)\in\mathcal{L}} x_{kj}, \ \forall k \in \mathcal{C} & \text{(balance cons.)} \\
 & 0 \leq x_{ij} \leq U_{ij}, \quad \forall i,j \in \mathcal{C} : (i,j) \in \mathcal{L} & \text{(bound cons.)}
\end{array}
$$