Topology Design of Cloaking Devices/Scatterers

Bilinear terms, wu, in PDE constraints ...

$$(P)\begin{cases} \underset{u,w}{\text{minimize}} & J(u) = \frac{1}{2}\|u + u_0\|^2_{2,\Omega_0} \\ \text{subject to} & -\Delta u - k_0^2(1+qw)u = k_0^2 q w u_0 \quad \text{in } \Omega \\ & \frac{\partial u}{\partial n} - ik_0 u = 0 \quad \text{on } \partial\Omega \\ & w \in \{0,1\} \quad \text{in } \hat{\Omega}. \end{cases}$$

... lead to nonconvex discretized problem

Exercise 1: Convexify (P)!

Derive the (4) McCormick under/over estimators for $z = wu$ for bounds $0 \le w \le 1,\ L \le u \le U$, e.g., $(1-w) \ge 0,\ (U-u) \ge 0$

$$\Rightarrow (1-w)(U-u) \ge 0 \quad \Rightarrow \quad U - Uw - u + z \ge 0.$$

How can we obtain bounds on the state u (for a fixed mesh)?

Control Regularization: Not All Norms Are Equal

Consider Poisson with Distributed Control, [OPTPDE, 2014]

$$
\begin{cases}
\underset{u,w}{\text{minimize}} & \|u - u_d\|^2_{L^2(\Omega)} + \int_\Gamma e_\Gamma \, u \, ds + \alpha \, \|w\|_{L^x} \\
\text{subject to} & -\Delta u + u = w + e_\Omega \quad \text{in } \Omega \\
& \frac{\partial u}{\partial n} = 0 \text{ on boundary } \Gamma \\
& w(x,y) \in \{0,1\}
\end{cases}
$$

Which regularizer? L^1 or L^2 for control $w(x,y) \in \{0,1\}$?

Exercise 2: Run AMPL model `MotherProb.mod` on NEOS

Show that L^1 or L^2 equivalent for $w(x,y) \in \{0,1\}$.
Which norm works better, and why?

OPTPDE (2014).
OPTPDE — a collection of problems in PDE-constrained optimization.
http://www.optpde.net.